

MHV amplitudes in $\mathcal{N} = 2$ SQCD and in $\mathcal{N} = 4$ SYM at one-loop

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ABSTRACT: Using four-dimensional unitarity and MHV-rules we calculate the one-loop MHV amplitudes with all external particles in the adjoint representation for $\mathcal{N} = 2$ supersymmetric QCD with N_f fundamental flavours. We start by considering such amplitudes in the superconformal $\mathcal{N} = 4$ gauge theory where the $\mathcal{N} = 4$ supersymmetric Ward identities (SWI) guarantee that all MHV amplitudes for all types of external particles are given by the corresponding tree-level result times a universal helicity- and particle-type-independent contribution. In $\mathcal{N} = 2$ SQCD the MHV amplitudes differ from those for $\mathcal{N} = 4$ for general values of N_f and N_c . However, for $N_f = 2N_c$ where the $\mathcal{N} = 2$ SQCD is conformal, the $\mathcal{N} = 2$ MHV amplitudes (with all external particles in the adjoint representation) are identical to the $\mathcal{N} = 4$ results. This factorisation at one-loop motivates us to pose a question if there may be a BDS-like factorisation for these amplitudes which also holds at higher orders of perturbation theory in superconformal $\mathcal{N} = 2$ theory.

KEYWORDS: Supersymmetric gauge theory, Extended Supersymmetry, NLO Computations, QCD.

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1. Introduction

The last few years have seen some remarkable progress in our understanding of the structure of gluonic scattering amplitudes in maximally supersymmetric $\mathcal{N}=4$ gauge theory (MSYM).

On the one hand there is the remarkable proposal of Bern, Dixon and Smirnov [1] for maximal helicity violating (MHV) n -point amplitudes to all orders in planar perturbation theory. Their formula has been confirmed for $n = 4$ -point amplitudes at three loops [2, 1], and for $n = 5$ at two loops in [3]. In fact, there are strong reasons [4, 5] to believe that the BDS formula for $n = 4$ and $n = 5$ is correct to all orders in planar perturbation theory. It is also known that the BDS conjecture does not agree with the explicit two-loop calculation of a 6-point MHV amplitude [6], and has to be corrected by an as yet unknown remainder function of certain dual-space-conformally-invariant ratios of kinematic invariants [6–8].

On the other hand, but also related to this, there is emerging evidence for a novel duality relation between the planar MHV amplitudes and the light-like perturbative Wilson loops proposed by Drummond, Korchemsky and Sokatchev [9] and further developed in refs. [10, 4, 5, 7, 8]. As already mentioned, recently a computation of the “parity even” part of the six-gluon MHV amplitude [6] in $\mathcal{N} = 4$ has shown that the BDS ansatz for MHV amplitudes does fail for $n = 6$. However, a numerical comparison [6, 8] with the corresponding hexagonal Wilson loop shows that the MHV-amplitude/Wilson-loop duality is correct at two loops and $n = 6$. This is a remarkable result.

There is also another route to verify the exponentiated structure of the gauge theory amplitudes implied by the BDS formula. In ref. [11] Alday and Maldacena gave a string theory prescription for computing planar $\mathcal{N} = 4$ amplitudes at strong coupling using the AdS/CFT correspondence. These amplitudes are determined by a certain classical string

solution and contain a universal exponential factor involving the action of the classical string. For 4-point amplitudes this classical action was calculated in [11] and matched with the BDS prediction.¹ More generally there is now a string theory explanation for why planar amplitudes exponentiate. Remarkably, the same exponentiation is expected to hold not only for the MHV, but also for the non-MHV amplitudes [12] — though for the latter case the exponentiation can only occur in the strong coupling limit (and does not hold in the weakly coupled perturbation theory).

It should be extremely interesting to attempt to generalise these results to theories with less than maximal amount of supersymmetry and, in these cases, also to allow for matter fields in the external states. Of course, one cannot hope for miracles, in order to preserve the beautiful structure which has emerged in the $\mathcal{N} = 4$ settings, the less supersymmetric theories should probably maintain some powerful feature in common with $\mathcal{N} = 4$.

The main goal of this paper is to investigate planar MHV amplitudes in $\mathcal{N} = 2$ supersymmetric QCD (SQCD) in the conformal phase at one-loop. Gauge theories with $\mathcal{N} = 2$ supersymmetry have been studied in great detail especially in the context of the Seiberg-Witten theory [14]. The scattering amplitudes in $\mathcal{N} = 2$ gauge theory, however, have not been analysed in detail so far. We note that recent papers [15–17] discuss $\mathcal{N} = 2$ scattering amplitudes in string theory settings.

The use of four-dimensional on-shell techniques, originally pioneered by Bern et al [18–20] in the mid-90's has lead to a vast reduction in the complexity of one-loop calculations. The use of gauge-invariant physical amplitudes (at tree level) as building blocks means that simplifications due to the large cancellation of Feynman diagrams occur in the preliminary stages of the calculation, rather than the latter. The unitarity method sews together four-dimensional tree-level amplitudes and, using unitarity to reconstruct the (poly)logarithmic cut constructible part of the amplitude, successfully reproduces the coefficients of the cut-constructible pieces of a one-loop amplitude. This has extensive uses in supersymmetric Yang-Mills theories, which are cut-constructible i.e. the whole amplitude can be reconstructed from knowledge of its discontinuities.

The tree-level amplitudes appearing in the cuts are efficiently determined by the MHV-rules method of Cachazo, Svrcek and Witten [21]. The rules hinge on the realisation that MHV tree amplitudes can act as vertices contributing to amplitudes with any number of negative helicity gluons [21] and all other fields present in a theory [22, 23]. Brandhuber, Spence and Travaglini (BST) [24–26] then showed how the MHV rules can be used at one-loop for the calculation of n -point gluonic MHV amplitudes.

In this paper, we will use the four-dimensional unitarity method of Bern, Dixon, Dunbar and Kosower [18–20] in concert with the one-loop MHV-rules formulation of Brandhuber, Spence and Travaglini [24] to calculate MHV amplitudes in both $\mathcal{N} = 4$ SYM and $\mathcal{N} = 2$ SQCD with N_f flavours. We check that amplitudes with external vector, scalar and fermionic external legs satisfy the SWI when all particles are in the adjoint representation. At one-loop, and for general values of N_f , we find that the amplitudes are cut-constructible and different from those in $\mathcal{N} = 4$ SYM, by an amount proportional to the result for a

¹Similar calculations for larger numbers of gluons [13] are incompatible with the BDS ansatz.

chiral $\mathcal{N} = 1$ multiplet. However, when $N_f = 2N_c$ and the SQCD becomes superconformal, all MHV amplitudes in $\mathcal{N} = 2$ with all external particles in the adjoint representation coincide with those in $\mathcal{N} = 4$.

The paper is organised as follows. In section 2 we will specify the complete set of component MHV amplitudes for the $\mathcal{N} = 2$ SQCD with N_f fundamental flavours. We also give a prescription how to relate the $\mathcal{N} = 4$ to the $\mathcal{N} = 2$ degrees of freedom. In section 3 we calculate the MHV amplitude for external adjoint particles in the $\mathcal{N} = 4$ and in the $\mathcal{N} = 2$ theory for general values of N_f and N_c . Our main result is that the $\mathcal{N} = 2$ MHV amplitudes agree with the corresponding $\mathcal{N} = 4$ results — but only in the superconformal limit when $N_f = 2N_c$.

A selection of very recent papers further discusses the MHV rules at one-loop in non-supersymmetric theories [27, 28], in $\mathcal{N} = 4$ SYM [29], in $\mathcal{N} = 8$ supergravity [30] and the universal infrared behavior in conformal gauge theories [31].

2. MHV amplitudes and supersymmetry

The simplest MHV amplitude is the n -gluon amplitude with two negative-helicity and $n-2$ positive-helicity gluons. The full set of n -point MHV amplitudes in MSYM is formed by all possible superpartners of the MHV amplitude with only gluons on the external lines.

Supersymmetric Ward identities (SWI) [32] which relate MHV amplitudes with different external lines follow from the susy algebra

$$\begin{aligned} [Q(\eta), \lambda^+(k)] &= -\theta \langle \eta k \rangle g^+(k), & [Q(\eta), \lambda^-(k)] &= +\theta [\eta k] g^-(k), \\ [Q(\eta), g^-(k)] &= +\theta \langle \eta k \rangle \lambda^-(k), & [Q(\eta), g^+(k)] &= -\theta [\eta k] \lambda^+(k). \end{aligned} \quad (2.1)$$

Here, g^\pm denote the helicity states of gluons and λ^\pm represents the gluinos of the ordinary SYM. As usual, instead of using the anticommuting spinor supercharge, we have contracted it with a commuting reference spinor η and multiplied it by a Grassmann number θ . This defines a commuting singlet operator $Q(\eta)$. The anticommuting parameter θ cancels from the relevant expressions for the amplitudes.

In order to relate all MHV amplitudes of the $\mathcal{N} = 4$ theory to each other [33] one needs to generalise the $\mathcal{N} = 1$ susy algebra (2.1) to $\mathcal{N} \geq 1$ theories. The $\mathcal{N} = 4$ susy relations were written down in [23, 34] and read:

$$[Q^A(\eta), g^+(k)] = -\theta_A [\eta k] \lambda^{+A}(k), \quad (2.2a)$$

$$[Q^A(\eta), \lambda^{+B}(k)] = -\delta^{AB} \theta_A \langle \eta k \rangle g^+(k) - \theta_A [\eta k] \phi^{AB}, \quad (2.2b)$$

$$[Q^A(\eta), \bar{\phi}_{AB}(k)] = -\theta_A [\eta k] \lambda_{\bar{B}}^-(k), \quad (2.2c)$$

$$[Q_A(\eta), \phi^{AB}(k)] = \theta_A \langle \eta k \rangle \lambda^{+B}(k), \quad (2.2d)$$

$$[Q_A(\eta), \lambda_{\bar{B}}^-(k)] = \delta_{AB} \theta_A [\eta k] g^-(k) + \theta_A \langle \eta k \rangle \bar{\phi}_{AB}(k), \quad (2.2e)$$

$$[Q_A(\eta), g^-(k)] = \theta_A \langle \eta k \rangle \lambda_{\bar{A}}^-(k). \quad (2.2f)$$

Our conventions are the same as in (2.1), and it is understood that $Q_A = Q^A$ and there is no summation over A in (2.2c), (2.2d). For scalar fields of the $\mathcal{N} = 4$ SYM, we use the

SU(4)_R conventions

$$\bar{\phi}_{AB} = \frac{1}{2} \epsilon_{ABCD} \phi^{CD} = (\phi^{AB})^\dagger. \quad (2.3)$$

Relations (2.2a)–(2.2f) uniquely determine all MHV amplitudes in $\mathcal{N} = 4$ SYM in terms of the MHV amplitude with only gluons on the external lines. In other words, the MHV amplitudes in $\mathcal{N} = 4$ form a single equivalence class under the $\mathcal{N} = 4$ SWI. Proportionality relations between different MHV $\mathcal{N} = 4$ amplitudes are entirely determined at tree-level. A simple prescription for writing them all down was found in refs. [23, 34] and for reader's convenience we summarise it in the appendix. (Another equivalent prescription was obtained more recently in [35]).

This simple general structure of MHV amplitudes in general does not hold at loop level for non-maximally supersymmetric theories. For example, in the $\mathcal{N} = 2$ SQCD there are a few separate equivalence classes, each characterised by the number of pairs of (anti)-fundamental fields present in the external states. There can be none, one or two such pairs for MHV amplitudes (and in addition in the latter case there is a technical subtlety caused by the fact that the two pairs can be of the same or of different flavours.) $\mathcal{N} = 2$ supersymmetry relates the MHV amplitudes within each class, but in the absence of additional supercharges, the different classes are not related.

The susy Ward identities and the resulting list of MHV amplitudes in $\mathcal{N} = 4$ (see refs. [23, 34]),²

$$\begin{aligned} & A_n(g^-, g^-), \quad A_n(g^-, \lambda_A^-, \lambda^{A+}), \quad A_n(\lambda_A^-, \lambda_B^-, \lambda^{A+}, \lambda^{B+}), \\ & A_n(g^-, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+}), \quad A_n(\lambda_A^-, \lambda^{A+}, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+}), \\ & A_n(\lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+}, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+}), \\ & A_n(\bar{\phi}_{AB}, \lambda^{A+}, \lambda^{B+}, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+}), \\ & A_n(g^-, \bar{\phi}_{AB}, \phi^{AB}), \quad A_n(g^-, \bar{\phi}_{AB}, \lambda^{A+}, \lambda^{B+}), \quad A_n(\lambda_A^-, \lambda_B^-, \phi^{AB}), \\ & A_n(\lambda_A^-, \phi^{BC}, \bar{\phi}_{BC}, \lambda^{A+}), \quad A_n(\lambda_A^-, \phi^{AB}, \bar{\phi}_{BC}, \lambda^{C+}), \quad A_n(\lambda_A^-, \bar{\phi}_{BC}, \lambda^{A+}, \lambda^{B+}, \lambda^{C+}), \\ & A_n(\bar{\phi}, \phi, \bar{\phi}, \phi), \quad A_n(\bar{\phi}, \phi, \bar{\phi}, \lambda^+, \lambda^+), \quad A_n(\bar{\phi}, \bar{\phi}, \lambda^+, \lambda^+, \lambda^+, \lambda^+), \end{aligned} \quad (2.4)$$

are most conveniently written using the SU(4)_R labelling conventions for scalars (2.3). However, in order to relate the MHV amplitudes above to those in $\mathcal{N} = 2$ SQCD, it is more appropriate to use $\mathcal{N} = 1$ supermultiplets, so that the $\mathcal{N} = 4$ theory contains one vector, V , and three adjoint chiral multiplets, Φ_1 , Φ_2 and Φ_3 . Similarly, the $\mathcal{N} = 2$ theory is described in terms of V , an adjoint chiral multiplet Φ , and N_f pairs of chiral fundamental (anti-fundamental) Q_f (\tilde{Q}_f) multiplets.

²In eqs. (2.4) we do not show positive-helicity gluons, and we do not distinguish between the different particle orderings in the amplitudes.

The $\mathcal{N} = 4$ scalars in the $SU(4)_R$, $SO(6)_R$ and $\mathcal{N} = 1$ language can be related as follows,

$$\phi^{12} \equiv \bar{\phi}_{34} = \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2) = \Phi_1, \quad (2.5)$$

$$\phi^{31} \equiv \bar{\phi}_{24} = \frac{1}{\sqrt{2}}(\phi^3 + i\phi^4) = \Phi_2, \quad (2.6)$$

$$\phi^{23} \equiv \bar{\phi}_{14} = \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6) = \Phi_3. \quad (2.7)$$

In components we have,

$$V = \begin{pmatrix} g^\pm \\ \lambda_1^\pm \end{pmatrix}, \quad \Phi_1 = \begin{pmatrix} \phi^{12} \\ \lambda_2^\pm \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi^{31} \\ \lambda_3^\pm \end{pmatrix}, \quad \Phi_3 = \begin{pmatrix} \phi^{23} \\ \lambda_4^\pm \end{pmatrix}. \quad (2.8)$$

In the above equations the first $\mathcal{N} = 1$ supersymmetry acts vertically within each column, while the second, third and fourth supersymmetry interchanges bosons of the first column with fermions of the second, third and fourth ones and so on. For $\mathcal{N} = 2$ SQCD we can make the following identification:

$$V = \begin{pmatrix} g^\pm \\ \lambda_1^\pm \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi^{12} \\ \lambda_2^\pm \end{pmatrix}, \quad Q_f = \begin{pmatrix} \phi^{31} \\ \lambda_3^\pm \end{pmatrix}, \quad \tilde{Q}_f = \begin{pmatrix} \phi^{23} \\ \lambda_4^\pm \end{pmatrix}. \quad (2.9)$$

The $\mathcal{N} = 4$ supersymmetry is now broken to $\mathcal{N} = 2$ since (anti)-fundamental fields cannot be exchanged with adjoint ones. Nevertheless, when working with primitive parts of the colour-ordered amplitudes there are two statements one can make for $\mathcal{N} = 2$ MHV's:

- (1) the list of MHV amplitudes is the same as in (2.4) with the substitutions (2.9);
- (2) the tree-level $\mathcal{N} = 2$ MHV amplitudes are the same as for $\mathcal{N} = 4$. This, however is in general no longer the case beyond the tree approximation.

In the appendix we list those tree-level MHV amplitudes which are needed for the one-loop calculations in subsequent sections.

2.1 The BST method for one-loop MHV amplitudes

In the BST approach [24] a generic diagram can be written:

$$\mathcal{D} = \frac{1}{(2\pi)^4} \int \frac{d^4 L_1}{L_1^2} \frac{d^4 L_2}{L_2^2} \delta^{(4)}(L_1 - L_2 - P) A_L(l_1, -P, -l_2) A_R(l_2, P, -l_1) \quad (2.10)$$

where $A_{L(R)}$ are the amplitudes for the left(right) vertices and P is the sum of momenta incoming to the right hand amplitude. The key step in the evaluation of this expression is to re-write the integration measure as an integral over the on-shell degrees of freedom and a separate integral over the complex variable z [24]:

$$\begin{aligned} \frac{d^4 L_1}{L_1^2} \frac{d^4 L_2}{L_2^2} &= (4i)^2 \frac{dz_1}{z_1} \frac{dz_2}{z_2} d^4 l_1 d^4 l_2 \delta^{(+)}(l_1^2) \delta^{(+)}(l_2^2) \\ &= (4i)^2 \frac{2dzdz'}{(z-z')(z+z')} d^4 l_1 d^4 l_2 \delta^{(+)}(l_1^2) \delta^{(+)}(l_2^2), \end{aligned} \quad (2.11)$$

where $z = z_1 - z_2$ and $z' = z_1 + z_2$. The integrand can only depend on z, z' through the momentum conserving delta function,

$$\delta^{(4)}(L_1 - L_2 - P) = \delta^{(4)}(l_1 - l_2 - P + z\eta) = \delta^{(4)}(l_1 - l_2 - \widehat{P}), \quad (2.12)$$

where $\widehat{P} = P - z\eta$. This means that the integral over z' can be performed so that,

$$\begin{aligned} \mathcal{D} &= \frac{(4i)^2 2\pi i}{(2\pi)^4} \int \frac{dz}{z} \int d^4 l_1 d^4 l_2 \delta^{(+)}(l_1^2) \delta^{(+)}(l_2^2) \delta^{(4)}(l_1 - l_2 - \widehat{P}) A_L(l_1, -P, -l_2) A_R(l_2, P, -l_1) \\ &= (4i)^2 2\pi i \int \frac{dz}{z} \int d\text{LIPS}^{(4)}(-l_1, l_2, \widehat{P}) A_L(l_1, -P, -l_2) A_R(l_2, P, -l_1), \end{aligned} \quad (2.13)$$

where,

$$d\text{LIPS}^{(4)}(-l_1, l_2, \widehat{P}) = \frac{1}{(2\pi)^4} d^4 l_1 d^4 l_2 \delta^{(+)}(l_1^2) \delta^{(+)}(l_2^2) \delta^{(4)}(l_1 - l_2 - \widehat{P}) \quad (2.14)$$

The phase space integral is regulated using dimensional regularisation. Tensor integrals arising from the product of tree amplitudes can be reduced to scalar integrals either by using spinor algebra or standard Passarino-Veltman reduction. The remaining scalar integrals have been evaluated previously by van Neerven [36].

At this point, one has obtained the discontinuity, or imaginary part, of the amplitude. However, by making a change of variables the final integration over the z variable can be cast as a dispersion integral

$$\frac{dz}{z} = \frac{d(\widehat{P})^2}{\widehat{P}^2 - P^2} \quad (2.15)$$

that re-constructs the full (cut-constructible part of the) amplitude. So far successful applications of this method include the calculation of the n -point pure gluon MHV amplitudes in $\mathcal{N} = 4$, $\mathcal{N} = 1$ and $\mathcal{N} = 0$ [24–26] and the non-supersymmetric n -point ϕ -MHV amplitudes [27, 28].

3. MHV amplitudes with n external gluons $A_n^{(1)}(g_1^-, \dots, g_m^-, \dots, n^+)$

In this section we shall calculate the one-loop corrections to the all-gluon MHV amplitude in both $\mathcal{N} = 4$ and $\mathcal{N} = 2$ supersymmetric QCD. For the maximally supersymmetric $\mathcal{N} = 4$ SYM these amplitudes are well-known. Our goal however is to apply the MHV-rules approach of [24] to the $\mathcal{N} = 2$ case.

The MHV-graphs contributing to the one-loop gluonic amplitude $A_n^{(1)}(g_1^-, \dots, g_m^-, \dots, n^+)$ are shown in figure 1. There are two distinct types of diagram, labelled (a) and (b) which are distinguished by the helicity flow around the loop and therefore by the types of the particles that are allowed to circulate in the loop. The individual MHV diagrams in figure 1 are then summed over via

$$(a) \sum_{i=m}^{n-2} \sum_{j=i+2}^n + \sum_{i=1}^{m-3} \sum_{j=i+2}^{m-1} \quad \text{and} \quad (b) \sum_{i=n}^n \sum_{j=2}^{m-1} + \sum_{i=m+1}^{n-1} \sum_{j=1}^{m-1} + \sum_{i=m}^m \sum_{j=1}^{m-2}. \quad (3.1)$$

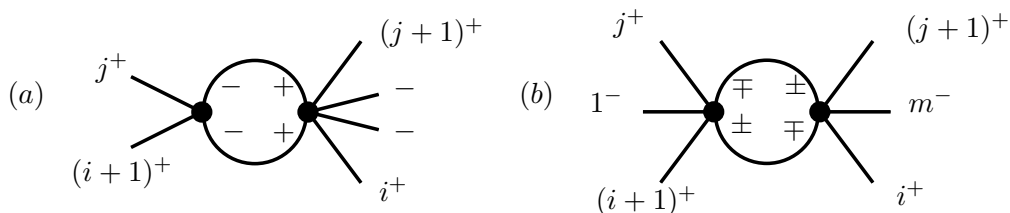


Figure 1: The MHV diagrams contributing to one-loop gluonic MHV amplitudes. In (a) only gluons circulate in the loop, while in (b) there are loop contributions from gluons, fermions and scalars. The momenta flowing across the cut is always $P_{j+1,i} = p_{j+1} + \dots + p_i$. The two diagrams differ in the locations of the negative helicity gluons with respect to i and j . In diagram (a) $i \geq m$ and $n \geq j$ as well as $i \geq 1$ and $m \geq j$, while for diagram (b) $(m-1) \geq j \geq 1$ and $n \geq i \geq m$

It is easy to see from the structure of tree-level MHV vertices that in figure 1(a), only gluons can circulate in the loop. For this reason these contributions are identical in $\mathcal{N} = 2$ and in $\mathcal{N} = 4$, and indeed for any theory involving gluons. On the other hand, the MHV graphs in figure 1(b) do receive contributions from gluons, fermions and scalars propagating in the loop, and as such differ in theories with different numbers of supercharges.

3.1 Contributions of the graph in figure 1(a)

Contributions depicted in figure 1(a) are associated with a cut in the $s_{(j+1),i}$ channel and have an integrand of the form,

$$\begin{aligned}
 (A_L A_R)_{(j+1),i} &= \frac{\langle \ell_1 \ell_2 \rangle^4}{\langle \ell_1(i+1) \rangle \dots \langle j \ell_2 \rangle \langle \ell_2 \ell_1 \rangle} \frac{\langle 1m \rangle^4}{\langle \ell_2(j+1) \rangle \dots \langle i \ell_1 \rangle \langle \ell_1 \ell_2 \rangle} \\
 &= \frac{\langle 1m \rangle^4}{\langle 12 \rangle \dots \langle n1 \rangle} \widehat{\mathcal{G}}(i, i+1, j, j+1) \\
 &\equiv A_n^{(0)} \widehat{\mathcal{G}}(i, i+1, j, j+1)
 \end{aligned} \tag{3.2}$$

where we have defined [28]

$$\widehat{\mathcal{G}}(i, i+1, j, j+1) = \frac{\langle \ell_2 \ell_1 \rangle \langle i(i+1) \rangle \langle \ell_1 \ell_2 \rangle \langle j(j+1) \rangle}{\langle i \ell_1 \rangle \langle \ell_1(i+1) \rangle \langle j \ell_2 \rangle \langle \ell_2(j+1) \rangle}, \tag{3.3}$$

where as usual A_L and A_R denote the tree-level MHV vertices respectively on the left and on the right side of the cut. As already mentioned, these type (a) diagrams give identical contributions in $\mathcal{N} = 4$ and in all theories with a gluon.

3.2 Contributions of the graph in figure 1(b)

We now turn to diagrams of type (b) where there are three possible contributions - depending on whether gluons, fermions or scalars are circulating in the loop. For each of the species in the loop it is convenient to add together both helicity assignments in figure 1(b)

such that

$$(A_L A_R)_{(j+1),i}^{\text{gluons}} = \frac{\langle 1\ell_2 \rangle^4 \langle m\ell_1 \rangle^4 + \langle 1\ell_1 \rangle^4 \langle m\ell_2 \rangle^4}{\langle \ell_1(i+1) \rangle \cdots \langle j\ell_2 \rangle \langle \ell_2 \ell_1 \rangle \langle \ell_2(j+1) \rangle \cdots \langle i\ell_1 \rangle \langle \ell_1 \ell_2 \rangle}, \quad (3.4)$$

$$(A_L A_R)_{(j+1),i}^{\text{fermions}} = \frac{\langle 1\ell_1 \rangle \langle 1\ell_2 \rangle^3 \langle m\ell_2 \rangle \langle m\ell_1 \rangle^3 + \langle 1\ell_2 \rangle \langle 1\ell_1 \rangle^3 \langle m\ell_1 \rangle \langle m\ell_2 \rangle^3}{\langle \ell_1(i+1) \rangle \cdots \langle j\ell_2 \rangle \langle \ell_2 \ell_1 \rangle \langle \ell_2(j+1) \rangle \cdots \langle i\ell_1 \rangle \langle \ell_1 \ell_2 \rangle}, \quad (3.5)$$

$$(A_L A_R)_{(j+1),i}^{\text{scalars}} = \frac{2\langle 1\ell_1 \rangle^2 \langle 1\ell_2 \rangle^2 \langle m\ell_2 \rangle^2 \langle m\ell_1 \rangle^2}{\langle \ell_1(i+1) \rangle \cdots \langle j\ell_2 \rangle \langle \ell_2 \ell_1 \rangle \langle \ell_2(j+1) \rangle \cdots \langle i\ell_1 \rangle \langle \ell_1 \ell_2 \rangle}. \quad (3.6)$$

In each case, the denominators have the same structure as in the (a)-type diagrams and only the numerators vary depending on the particle types. We can exploit the Schouten identity

$$\langle 1\ell_2 \rangle \langle m\ell_1 \rangle - \langle 1\ell_1 \rangle \langle m\ell_2 \rangle + \langle 1m \rangle \langle \ell_1 \ell_2 \rangle = 0 \quad (3.7)$$

to rewrite each of the numerators into a simpler form,

$$\begin{aligned} \langle 1\ell_2 \rangle^4 \langle m\ell_1 \rangle^4 + \langle 1\ell_1 \rangle^4 \langle m\ell_2 \rangle^4 &= \langle 1m \rangle^4 \langle \ell_1 \ell_2 \rangle^4 \\ &\quad + 4\langle 1\ell_2 \rangle \langle m\ell_1 \rangle \langle 1\ell_1 \rangle \langle m\ell_2 \rangle \langle 1m \rangle^2 \langle \ell_1 \ell_2 \rangle^2 \\ &\quad + 2\langle 1\ell_2 \rangle^2 \langle m\ell_1 \rangle^2 \langle 1\ell_1 \rangle^2 \langle m\ell_2 \rangle^2, \end{aligned} \quad (3.8)$$

and,

$$\begin{aligned} \langle 1\ell_1 \rangle \langle 1\ell_2 \rangle^3 \langle m\ell_2 \rangle \langle m\ell_1 \rangle^3 + \langle 1\ell_2 \rangle \langle 1\ell_1 \rangle^3 \langle m\ell_1 \rangle \langle m\ell_2 \rangle^3 &= \langle 1\ell_2 \rangle \langle m\ell_1 \rangle \langle 1\ell_1 \rangle \langle m\ell_2 \rangle \langle 1m \rangle^2 \langle \ell_1 \ell_2 \rangle^2 \\ &\quad + 2\langle 1\ell_2 \rangle^2 \langle m\ell_1 \rangle^2 \langle 1\ell_1 \rangle^2 \langle m\ell_2 \rangle^2. \end{aligned} \quad (3.9)$$

We see that the first term on the r.h.s. of eq. (3.8) corresponds to an (a)-type gluonic contribution which we will label as G , while the third term looks like the scalar contribution of eq. (3.6) which we will denote as S . The fermion contribution can be separated into a scalar piece S and an additional contribution labelled by F . These three contributions are defined as

$$\begin{aligned} (A_L A_R)_{(j+1),i}^G &= \frac{\langle 1m \rangle^4 \langle \ell_1 \ell_2 \rangle^4}{\langle \ell_1(i+1) \rangle \cdots \langle j\ell_2 \rangle \langle \ell_2 \ell_1 \rangle \langle \ell_2(j+1) \rangle \cdots \langle i\ell_1 \rangle \langle \ell_1 \ell_2 \rangle} \\ &= A_n^{(0)} \widehat{\mathcal{G}}(i, i+1, j, j+1), \end{aligned} \quad (3.10)$$

$$\begin{aligned} (A_L A_R)_{(j+1),i}^F &= \frac{\langle 1\ell_2 \rangle \langle m\ell_1 \rangle \langle 1\ell_1 \rangle \langle m\ell_2 \rangle \langle 1m \rangle^2 \langle \ell_1 \ell_2 \rangle^2}{\langle \ell_1(i+1) \rangle \cdots \langle j\ell_2 \rangle \langle \ell_2 \ell_1 \rangle \langle \ell_2(j+1) \rangle \cdots \langle i\ell_1 \rangle \langle \ell_1 \ell_2 \rangle} \\ &= -A_n^{(0)} \widehat{\mathcal{F}}(i, i+1, j, j+1), \end{aligned} \quad (3.11)$$

$$\begin{aligned} (A_L A_R)_{(j+1),i}^S &= \frac{\langle 1\ell_1 \rangle^2 \langle 1\ell_2 \rangle^2 \langle m\ell_2 \rangle^2 \langle m\ell_1 \rangle^2}{\langle \ell_1(i+1) \rangle \cdots \langle j\ell_2 \rangle \langle \ell_2 \ell_1 \rangle \langle \ell_2(j+1) \rangle \cdots \langle i\ell_1 \rangle \langle \ell_1 \ell_2 \rangle} \\ &= -A_n^{(0)} \widehat{\mathcal{S}}(i, i+1, j, j+1), \end{aligned} \quad (3.12)$$

with $\widehat{\mathcal{G}}(i, i+1, j, j+1)$ being given in eq. (3.3) and,

$$\widehat{\mathcal{F}}(i, i+1, j, j+1) = \frac{\langle i(i+1) \rangle \langle j(j+1) \rangle \langle 1\ell_1 \rangle \langle m\ell_1 \rangle \langle 1\ell_2 \rangle \langle m\ell_2 \rangle}{\langle 1m \rangle^2 \langle i\ell_1 \rangle \langle \ell_1(i+1) \rangle \langle j\ell_2 \rangle \langle \ell_2(j+1) \rangle}, \quad (3.13)$$

$$\widehat{\mathcal{S}}(i, i+1, j, j+1) = \frac{\langle i(i+1) \rangle \langle j(j+1) \rangle \langle 1\ell_1 \rangle^2 \langle m\ell_1 \rangle^2 \langle 1\ell_2 \rangle^2 \langle m\ell_2 \rangle^2}{\langle 1m \rangle^4 \langle \ell_1 \ell_2 \rangle^2 \langle i\ell_1 \rangle \langle \ell_1(i+1) \rangle \langle j\ell_2 \rangle \langle \ell_2(j+1) \rangle}. \quad (3.14)$$

$\widehat{\mathcal{G}}$, $\widehat{\mathcal{F}}$ and $\widehat{\mathcal{S}}$ are the basis functions of ref. [28]. We observe that $\widehat{\mathcal{G}}$ and $\widehat{\mathcal{F}}$ are completely cut-constructible, whilst $\widehat{\mathcal{S}}$ contains terms which arise from the reduction of third and second rank tensor triangles. These contain spurious singularities, for which we must include additional rational terms to “complete” the amplitude. We further note that for amplitudes involving only gluons, $\widehat{\mathcal{G}}$ produces only one- and two-mass easy box functions.

We now need to restore the particle multiplicities. For the $\mathcal{N} = 4$ MSYM case with four adjoint fermions and three adjoint scalars we have,

$$\begin{aligned}
 (A_L A_R)_{(j+1),i}^{\mathcal{N}=4} &= (A_L A_R)_{(j+1),i}^{\text{gluons}} - 4 (A_L A_R)_{(j+1),i}^{\text{fermions}} + 3 (A_L A_R)_{(j+1),i}^{\text{scalars}} \\
 &= \left((A_L A_R)_{(j+1),i}^{\text{G}} + 4 (A_L A_R)_{(j+1),i}^{\text{F}} + 2 (A_L A_R)_{(j+1),i}^{\text{S}} \right) \\
 &\quad - 4 \left((A_L A_R)_{(j+1),i}^{\text{F}} + 2 (A_L A_R)_{(j+1),i}^{\text{S}} \right) \\
 &\quad + 3 \left(2 (A_L A_R)_{(j+1),i}^{\text{S}} \right) \\
 &\equiv A_n^{(0)} \widehat{\mathcal{G}}(i, i+1, j, j+1).
 \end{aligned} \tag{3.15}$$

This is the key result for $\mathcal{N} = 4$ MSYM one-loop amplitudes - all cuts yield the same “gluonic” contribution independently of the particles circulating around the loop. As we will see, the same result is obtained independently of the choice of external particles as required by the SWI.

For the $\mathcal{N} = 2$ SQCD case with N_f (anti)-fundamental flavours \tilde{Q}_f, Q_f the degrees of freedom propagating in the loop come from the $\mathcal{N} = 1$ vector superfield V , from the adjoint chiral $\mathcal{N} = 1$ superfield Φ , and from N_f pairs of Q_f and \tilde{Q}_f . In components we have for V :

$$\begin{aligned}
 (A_L A_R)_{(j+1),i}^{\mathcal{N}=1,V} &= (A_L A_R)_{(j+1),i}^{\text{gluons}} - (A_L A_R)_{(j+1),i}^{\text{fermions}} \\
 &= \left((A_L A_R)_{(j+1),i}^{\text{G}} + 4 (A_L A_R)_{(j+1),i}^{\text{F}} + 2 (A_L A_R)_{(j+1),i}^{\text{S}} \right) \\
 &\quad - \left((A_L A_R)_{(j+1),i}^{\text{F}} + 2 (A_L A_R)_{(j+1),i}^{\text{S}} \right) \\
 &= (A_L A_R)_{(j+1),i}^{\text{G}} + 3 (A_L A_R)_{(j+1),i}^{\text{F}},
 \end{aligned} \tag{3.16}$$

for Φ :

$$\begin{aligned}
 (A_L A_R)_{(j+1),i}^{\mathcal{N}=1,\Phi} &= (A_L A_R)_{(j+1),i}^{\text{scalars}} - (A_L A_R)_{(j+1),i}^{\text{fermions}} \\
 &= 2 (A_L A_R)_{(j+1),i}^{\text{S}} - \left((A_L A_R)_{(j+1),i}^{\text{F}} + 2 (A_L A_R)_{(j+1),i}^{\text{S}} \right) \\
 &= - (A_L A_R)_{(j+1),i}^{\text{F}},
 \end{aligned} \tag{3.17}$$

and for each of Q_f and \tilde{Q}_f :

$$(A_L A_R)_{(j+1),i}^{\mathcal{N}=1,Q_f(\tilde{Q}_f)} = -\frac{1}{2N_c} (A_L A_R)_{(j+1),i}^{\text{F}}. \tag{3.18}$$

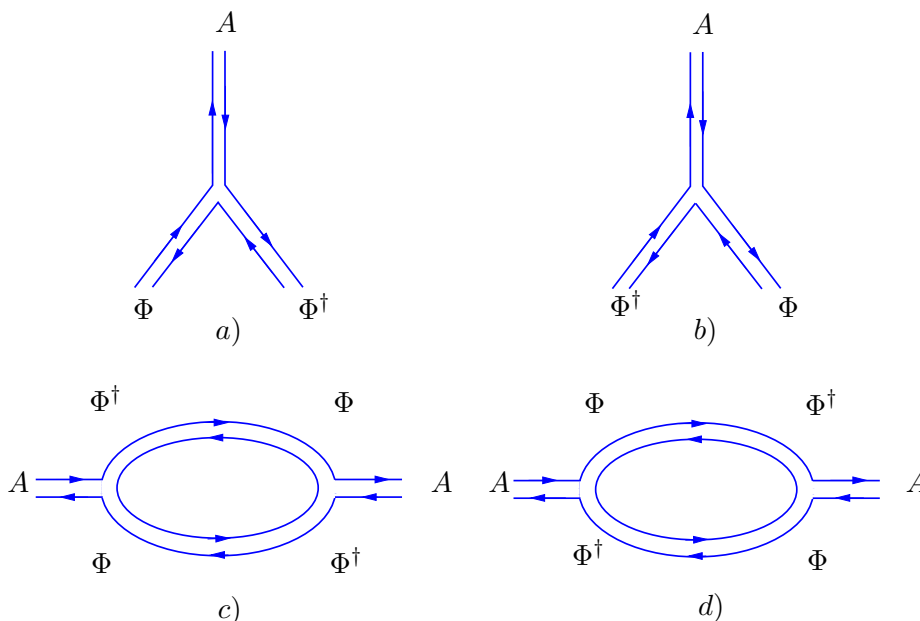


Figure 2: (a) and (b) show two distinct interaction vertices of gluons with matter fields in the adjoint representation using the 't Hooft double-line colour flow representation. Figures (c) and (d) represent the two resulting matter-field contributions in the loop with external gluons.

In the last equation we used the fact that the adjoint and the fundamental chiral multiplets propagating in the loop in figure 1(b) contribute equally up to the normalisation factor $1/(2N_c)$. This is of course analogous to the computation of the one-loop b_0 coefficient of the beta function in SQCD where each of the fundamental $Q_f(\tilde{Q}_f)$ superfields contributes with a weight of $-1/2$ while the adjoint Φ multiplet contributes a factor of $-N_c$. The factor of $1/2$ arises from the fact that the commutator in the covariant derivative for the adjoint matter fields contains two terms, hence there are two differently ordered vertices in figures 2(a) and (b) compared to a single fundamental vertex in figure 3(a). Thus the sum of contributions in figures 2(c) and (d) is equal to $2N_c$ times the contribution in figure 3(b), where N_c arises from the inner colour loop in figures 2(c) and (d).

In summary, the total contribution in $\mathcal{N} = 2$ SQCD with N_f flavours (for all-external-gluon one-loop MHV amplitudes) is

$$(A_L A_R)_{(j+1),i}^{\mathcal{N}=2} = (A_L A_R)_{(j+1),i}^G + 2 \left(1 - \frac{N_f}{2N_c} \right) (A_L A_R)_{(j+1),i}^F. \quad (3.19)$$

This is to be compared with the $\mathcal{N} = 4$ SYM contribution which is simply

$$(A_L A_R)_{(j+1),i}^{\mathcal{N}=4} = (A_L A_R)_{(j+1),i}^G. \quad (3.20)$$

We can make several remarks concerning the $\mathcal{N} = 2$ SQCD case. Firstly as expected the amplitude is cut-constructible, since the absence of scalar terms ensures there is no need for cut-completing terms. Secondly, for the superconformal case $N_f = 2N_c$ there are

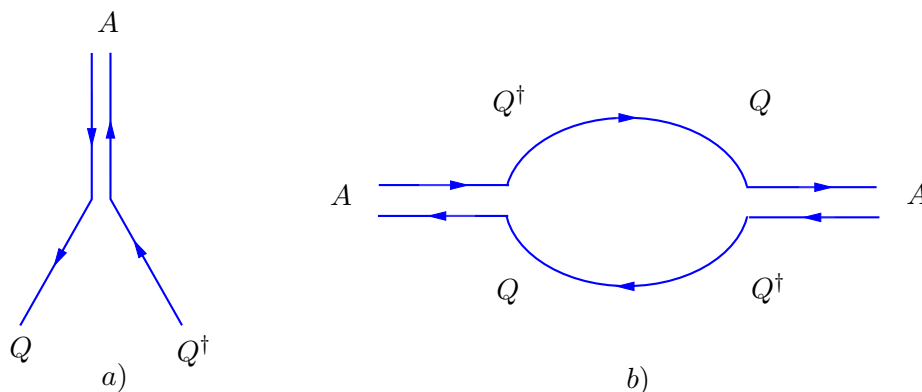


Figure 3: The interaction vertex for gluons with fundamental matter and the corresponding matter-field contribution to the loop.

no contributions from F terms, meaning that,

$$(A_L A_R)_{(j+1),i}^{\mathcal{N}=2} \Big|_{N_f=2N_c} = (A_L A_R)_{(j+1),i}^{\mathcal{N}=4}. \quad (3.21)$$

This equation demonstrating equality of n -gluon MHV amplitudes in two different super-conformal theories is one of our main results.

The total one-loop pure glue MHV amplitude in $\mathcal{N} = 2$ SQCD is given by,

$$\begin{aligned} A_n^{(1)}(1^-, \dots, m^-, \dots, n^+) \\ = c_\Gamma A_n^{(0)} \left(A_{n;1}^G(1, m) - 2 \left(1 - \frac{N_f}{2N_c} \right) A_{n;1}^F(1, m) \right) \end{aligned} \quad (3.22)$$

where the helicity independent function $A_{n;1}^G(1, m)$ is given by

$$A_{n;1}^G(1, m) = -\frac{1}{2} \sum_{i=1}^n F_4^{1m}(s_{i,i+2}; s_{i,i+1}, s_{i+1,i+2}) - \frac{1}{4} \sum_{i=1}^n \sum_{j=i+3}^{n+i-3} F_4^{2me}(s_{i,j}, s_{i+1,j-1}; s_{i+1,j}, s_{i,j-1}) \quad (3.23)$$

and the helicity dependent function $A_{n;1}^F(1, m)$ is

$$\begin{aligned} A_{n;1}^F(1, m) = & \sum_{i=m+1}^n \sum_{j=2}^{m-1} b_{1m}^{ij} F_{4F}^{2me}(s_{i,j}, s_{i-1,j+1}; s_{i-1,j}, s_{i,j+1}) \\ & - \sum_{i=2}^{m-1} \sum_{j=m}^n \frac{\text{tr}_-(1, P_{(i,j)}, i, m)}{s_{1m}^2} \mathcal{A}_{1m}^{ij} T_1(P_{(i+1,j)}, P_{(i,j)}) \\ & + \sum_{i=2}^m \sum_{j=m+1}^n \frac{\text{tr}_-(1, P_{(i,j-1)}, j, m)}{s_{1m}^2} \mathcal{A}_{1m}^{j(i-1)} T_1(P_{(i,j-1)}, P_{(i,j)}). \end{aligned} \quad (3.24)$$

Here we have introduced the shorthand notation

$$\text{tr}_-(abcd) = \langle ab \rangle [bc] \langle cd \rangle [da] \quad (3.25)$$

and the auxiliary functions,

$$\begin{aligned}
 b_{1m}^{ij} &= \frac{\text{tr}_-(1, i, j, m) \text{tr}_-(1, j, i, m)}{s_{ij}^2 s_{1m}^2}, \\
 \mathcal{A}_{1m}^{ij} &= \left(\frac{\text{tr}_-(1, i, j, m)}{s_{ij}} - (j \rightarrow j+1) \right).
 \end{aligned}
 \tag{3.26}$$

Note that b_{1m}^{ij} is symmetric under both $i \leftrightarrow j$ and $1 \leftrightarrow m$, while \mathcal{A}_{1m}^{ij} is antisymmetric under $1 \leftrightarrow m$. The function F_{4F}^{2me} is the finite pieces of the two mass easy box function (or the finite pieces of the one mass box function in the limit where one of the massive legs becomes massless). We define the triangle function $T_i(P, Q)$ as,

$$T_i(P, Q) = L_i(P, Q) = \frac{\log(P^2/Q^2)}{(P^2 - Q^2)^i} \quad P^2 \neq 0, \quad Q^2 \neq 0.
 \tag{3.27}$$

If one of the invariants becomes massless then the triangle function becomes the divergent function,

$$T_i(P, Q) \rightarrow (-1)^i \frac{1}{\epsilon} \frac{(-P^2)^{-\epsilon}}{(P^2)^i}, \quad Q^2 \rightarrow 0.
 \tag{3.28}$$

We note that in $\mathcal{N} = 4$ MSYM, the one-loop MHV amplitude is given by [18, 24]

$$A_n^{(1)}(1^-, \dots, m^-, \dots, n^+) = c_\Gamma A_n^{(0)} A_{n;1}^G(1, m),
 \tag{3.29}$$

while in the $\mathcal{N} = 1$ theory with a chiral multiplet [19, 25],

$$A_n^{(1)}(1^-, \dots, m^-, \dots, n^+) = c_\Gamma A_n^{(0)} A_{n;1}^F(1, m).
 \tag{3.30}$$

The one-loop amplitude for $\mathcal{N} = 2$ SQCD is thus a linear combination of the amplitudes for $\mathcal{N} = 4$ MSYM and $\mathcal{N} = 1$, which in the superconformal limit, collapses to the $\mathcal{N} = 4$ MSYM result.

We have explicitly checked that the amplitudes with external pairs of fermions and scalars (all in the adjoint representation) yield identical results (up to the tree-level factor) as expected by the SWI in both the $\mathcal{N} = 4$ MSYM and $\mathcal{N} = 2$ SQCD theories.

4. Summary

In this paper, we have computed the one-loop MHV amplitude in the $\mathcal{N} = 2$ supersymmetric QCD with N_f fundamental flavours. We have focussed on the case where all of the external particles are in the adjoint representation. Our main result is eq. (3.22) which shows that the $\mathcal{N} = 2$ amplitude is, for general values of N_f and N_c , simply a combination of the corresponding amplitudes in the $\mathcal{N} = 4$ and chiral $\mathcal{N} = 1$ supersymmetric gauge theories. When $N_f = 2N_c$, where the $\mathcal{N} = 2$ SQCD is conformal, the superconformal $\mathcal{N} = 2$ MHV amplitudes are identical to the $\mathcal{N} = 4$ results. This factorisation at one-loop leads us to pose a question if there may be a BDS-like iterative structure for these ‘‘adjoint’’ amplitudes at higher orders of perturbation theory in superconformal $\mathcal{N} = 2$ theory.

It still remains to study the one-loop MHV amplitudes in the $\mathcal{N} = 2$ SQCD theory with external particles in the fundamental representation. These amplitudes have a quite different colour structure, and we expect that there are distinct classes of MHV amplitudes in this case. Each class is characterised by a number of pairs of (anti)-fundamental external fields, there can be none, one or two such pairs for MHV amplitudes. $\mathcal{N} = 2$ supersymmetry relates the MHV amplitudes within each class, but in the absence of additional supercharges, the different classes are not related.

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A. MHV tree amplitudes

Here we summarise the general rule obtained in refs. [23, 34] for writing down the tree-level contributions for all the component MHV-amplitudes listed in (2.4). Following [37] this is done by first introducing the auxiliary anticommuting spinors η_a^A (here $A = 1, 2, 3, 4$ and $a = 1, 2$ is the spinor index) for each external leg. Each external leg i is then associated with a monomial in η_i following the rule,

$$g_i^- \sim \eta_i^1 \eta_i^2 \eta_i^3 \eta_i^4, \Lambda_1^- \sim -\eta_i^2 \eta_i^3 \eta_i^4, \phi_i^{AB} \sim \eta_i^A \eta_i^B, \Lambda_i^{A+} \sim \eta_i^A, g_i^+ \sim 1, \quad (\text{A.1})$$

with expressions for the remaining Λ_A^- with $A = 2, 3, 4$ written in the same manner as the expression for Λ_1^- in (A.1).

The MHV amplitudes are then obtained as follows:

1. For each amplitude in (2.4) substitute the fields by their η -expressions (A.1). There are precisely eight η 's for each MHV amplitude (in fact this, rather than the number of negative helicities, is the definition of MHV amplitudes).
2. Keeping track of the overall sign, rearrange the anticommuting η 's into a product of four pairs: $(\text{sign}) \times \eta_i^1 \eta_j^1 \eta_k^2 \eta_l^2 \eta_m^3 \eta_n^3 \eta_r^4 \eta_s^4$.
3. The amplitude is obtained by replacing each pair $\eta_i^A \eta_j^A$ by the spinor product $\langle i j \rangle$ and dividing by the usual denominator,

$$A_n = (\text{sign}) \times \frac{\langle i j \rangle \langle k l \rangle \langle m n \rangle \langle r s \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}. \quad (\text{A.2})$$

In this way one can immediately write down expressions for all component amplitudes in (2.4). It can be checked that these expressions are inter-related via $\mathcal{N} = 4$ susy Ward identities which follow from the $\mathcal{N} = 4$ susy algebra in eqs. (2.2a)–(2.2f).

The following tree amplitudes are useful in our calculation of $\mathcal{N} = 4$ MHV amplitudes at one-loop;

$$A_n(g_i^-, g_j^-) = \frac{\langle ij \rangle^4}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.3})$$

$$A_n(g_i^-, \lambda_A^-(k), \lambda^{A+}(j)) = \frac{\langle ik \rangle^3 \langle ij \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.4})$$

$$A_n(g_i^-, \lambda^{A+}(j), \lambda_A^-(k)) = -\frac{\langle ik \rangle^3 \langle ij \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.5})$$

$$A_n(g^-(i), \bar{\phi}_{AB}(j), \phi^{AB}(k)) = \frac{\langle ik \rangle^2 \langle ij \rangle^2}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.6})$$

where $\lambda_A^+(j)$ represents a positive helicity gluino with momenta p_j here $A, B = 1, 2, 3, 4$ are the four supersymmetric multiplets. the first two gluino case corresponds to $k < j$ whereas in the second case $j < k$. For the calculation of the one-loop corrections to the processes $A_n(g^-(i), \bar{\phi}_{AB}(j), \phi^{AB}(k))$ we need the following additional trees;

$$A_n(\lambda^{A+}(k), g^-(i), \bar{\phi}_{AB}(j), \lambda^{B+}(l)) = \frac{\langle ij \rangle^2 \langle il \rangle \langle ki \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.7})$$

$$A_n(\lambda_A^-(i), \phi^{AB}(k), \lambda_B^-(j)) = \frac{\langle ij \rangle^2 \langle ik \rangle \langle kj \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.8})$$

$$A_n(\lambda_A^-(i), \phi^{BC}(k), \bar{\phi}_{BC}(j), \lambda^{A+}(l)) = -\frac{\langle ik \rangle^2 \langle ij \rangle \langle jl \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}, \quad (\text{A.9})$$

$$A_n(\phi^{AB}(i), \bar{\phi}_{CD}(j), \phi^{CD}(k), \bar{\phi}_{AB}(l)) = \frac{\langle ik \rangle^2 \langle jl \rangle^2}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}. \quad (\text{A.10})$$

For the one-loop corrections to $A_n(g^-, \lambda_A^-, \lambda^{A+})$ we also need the four-fermion vertex,

$$A_n(\lambda_A^-(i), \lambda^{B+}(j), \lambda_B^-(k), \lambda^{A+}(l)) = -\frac{\langle ik \rangle^2 \langle kl \rangle \langle ij \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle},$$

$$A_n(\lambda^-(i), \lambda^+(j), \lambda^-(k), \lambda^+(l)) = \frac{\langle ik \rangle^3 \langle jl \rangle}{\prod_{\alpha=1}^n \langle \alpha \alpha + 1 \rangle}. \quad (\text{A.11})$$

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